Supplementary Handout

A Compliance Number Concept for Compliant Mechanisms, and Type Synthesis

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While much has been contributed to techniques for enumerating and identifying rigid-body mechanisms in the past decades, proportionally little has been accomplished in this regard in compliant mechanisms design. This paper deals primarily with identification and discussion of important kinematic properties of compliant mechanisms. To facilitate these appropriate terminology is developed at the very fundamental level. The conventional degrees-of-freedom concept for a rigid-body chain is briefly reviewed. It is then used to help define a compliance number (or degrees-of-compliance) concept for characterizing compliant mechanisms. Finally, a systematic and convenient approach is presented, enabling the type synthesis of this class of mechanisms.

Introduction

As defined by the authors in [1], the compliant mechanisms represents a class of mechanical systems which makes use of large-deformation flexible elements in design, as opposed to the use of only rigid-body members. The classification of compliant mechanisms, consequently, includes compliant linkages, cam-follower systems, gear trains, etc. We shall confine ourselves however, to the discussion of compliant linkages only, and compliant mechanisms here will denote compliant linkages. For more than a decade, a number of reseachers have studied the behavior of rigid-body linkages containing flexible parts. A discussion of early publications in this area may be found in [2].

Recently, a great deal of interest has been vested in the study of "kineto-elastodynamics" of mechanisms. This area of study pertains to the motion of mechanisms with consideration of elasticity and mass distribution in the links. Their effects in adding a vibratory component to the "rigid-body" mechanism response become quite significant at high speeds. In general, in these studies, such vibratory response is regarded as undesirable and detrimental to the health of machines that incorporate these mechanisms. Such a response tends to introduce positional errors in the mechanism, rendering its function inadequate, and reduce its life due to early failure from fatigue. A representative set of analytical and experimental references to such works may be found in [3].

Unlike the interest in mechanism flexibility in the foregoing discussion, designers sometimes advocate the incorporation of flexible members in mechanical systems to beneficial ends. However, due to a lack of systematic classification and understanding, the design of these types of mechanisms continues to rely upon the intuition and experience of the designer. Unlike categorizations of rigid-body mechanisms, e.g. [4], the classification of compliant mechanisms is made more difficult because of the very limited number of applications seen to date. This is particularly true when the compliance is of the distributed type in the links, as opposed to discrete compliance in the treatment of simple helical springs. As the design theory is better understood, and stronger and more resilient materials are developed, it is expected that the near future will bring a significant increase in the use of compliant mechanisms.

The synthesis and analysis of "flexible-link mechanisms" were first explored by Burns and Crossley [5]. They noted important differences between their analytical approach and rigid-body kinematic analysis. This is mainly due to the fact that the loading conditions of the compliant mechanism must now be considered in computing its displacements. Shoup and McLarnan [6, 7] applied the equations of the undulating and nodal elastica, and investigated the use of flexible-link devices in producing nonlinear force-deflection relationships. They also attempted to enumerate all possible single, closed-loop flexible-link mechanisms having lower pairs and a rigid-body degree of freedom less than unity. Sevak [8], and Sevak and McLarnan [9], used a nonlinear finite element method and an optimization technique to synthesize the above-mentioned flexible-link mechanisms for function generation. They observed a number of advantages in the use of flexible-link mechanisms over conventional linkages. Among them are lower manufacturing cost, simpler construction, no backlash and wear, nor the need for lubrication. Winter and Shoup [10] generated the coupler curves for some flexible-link mechanisms with one flexural member. This work is somewhat analogous to the classical Hrones and Nelson charts for rigid-body four-bar linkages. However, a significant difference is that these coupler curves [10] may change with the loading condition of the mechanism. Most recently, Eijk [11]

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presented in great detail a fairly complete discussion on the techniques of designing "plate-spring" mechanisms. The plate-spring mechanisms are generally simpler and more comprehensible in structure, with numerous industrial applications.

Most of the above-mentioned works emphasized development of numerical methods for mechanism analysis and synthesis. The purpose of this paper is to identify and discuss some important kinematic properties of compliant mechanisms. To facilitate this, appropriate nomenclature is developed. The degree-of-freedom concept for rigid-body mechanisms is briefly reviewed, and then used to help define a degree-of-compliance concept for compliant mechanisms. Finally, a convenient approach is presented to enable the type synthesis of compliant mechanisms.

Definitions and Properties Significant to Compliant Mechanisms

According to Reuleaux [12], a mechanism is "an assemblage of resistant bodies, connected by movable joints, to form a closed kinematic chain with one link fixed and having the purpose of transforming motion." The word "link" is used to designate the "resistant body" in the above definition. The joint, or the "movable" connection between the links, should ideally pose no resistance to the allowable motion between the links. If the relative motion of two links, connected at a joint, is an n-parameter motion, then this joint is said to possess n degrees of freedom. Thus, a joint always makes a positive contribution to the number of degrees of freedom of the mechanism.

Various types of rigid-body kinematic joints, and their properties, are well discussed in the literature, e.g., [12, 13]. For example, there are revolute, prism and screw joints, among the single-degree-of-freedom lower-pair joints. For simplicity, only lower pairs are considered here. Rigid-body links, when classified by their number of joints, are binary links, ternary links, quaternary links, etc. Binary links are the most basic rigid-body link type. All other rigid-body link types can be represented by their equivalent sets of binary links. Mruthyunjaya [14] investigated a method to transform any kinematic chain to a binary one. In this paper, only binary rigid-body links, represented as possessing two joints, will be treated in the various discussions.

Burns [2] argued that it is more complex to categorize parts in a compliant mechanism, since a part may sometimes behave like a link, and at other times like a joint. For this reason, he surmised that Grübler's criterion for evaluating the degrees of freedom will not work for such mechanisms. In previous works, if a flexible part was considered relatively short and slender, it was called a flexural joint, otherwise a flexible link. Thus, the flexural pivot, a well-developed and commercialized "plate-spring" mechanism (Fig. 1), is not a simple rigid-body joint. This device simulates a pin-jointed motion of two rigid-body links, joined together by orthogonally clamped flexible plates. It may be regarded as an elastica whose flexibility provides the freedom of rotational motion between the two links that are attached to it. In this paper, however, the flexural pivot and the attached links will be treated together as one compliant link.

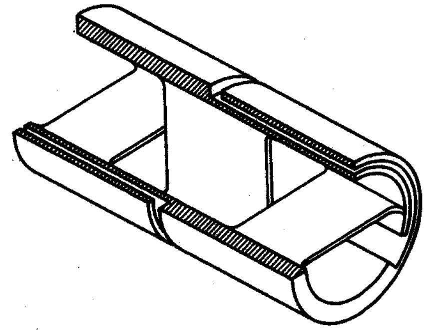

Fig. 1 A typical flexural pivot

The "rigidity" of a link at a given section is mathematically defined as the product EI, E representing the modulus of elasticity of the link material, and I its section moment of inertia. When this rigidity approaches infinity, the link is said to be rigid. On the other hand, if the link rigidity is low, contributing to non-negligible deformations under the applied loads, the link is referred to as a compliant or flexible link. Generally, a compliant link may undergo large delections. The construction of a compliant link need not be from isotropic or homogeneous materials; nor is the link required to have a uniform cross-sectional area. It may be of arbitrary shape, have rigid-body members joined with flexible ones, and may be branched. Figure 2 illustrates some compliant link types. In a compliant ternary link, when all three joints are not connected by a single, simple or compound binary compliant link, the double-branched configurations (Figs. 2(*g*) and 2(*h*)) result. A compliant mechanism may then be comprised of links and joints of the types discussed above. Thus, the compliant kinematic chain may constitute any of the following: an open chain or a closed one, a structure, a conventional rigid-body mechanism incorporating some discrete compliance, a flexible-link mechanism, or simply an elastica.

Suppose an input to a rigid-body mechanism is prescribed on a link, where there is no joint. This input would be transferred to a nearby joint, and expressed in terms of ap-

Nomenclature

- c_j = number of joints incorporating compliances in a kinematic chain, in type synthesis
- c_l = number of rigid-body links replaced by flexible ones in a chain, in type synthesis
- dx_J = x-directional degree of freedom (displacement) of joint J
- dy_J = y-directional degree of freedom (displacement) of joint J
- $d\theta_{iJ}$ = rotational degree of freedom (displacement) of joint J with respect to link i
- f_e = degrees of freedom of a compliant mechanism due to the flexible links only
- f_r = degrees of freedom of a compliant mechanism due to the rigid-body links only
- n_{ji} = number of i-degree-of-freedom rigid-body joints in a kinematic chain
- n_l = number of links in a chain
- n_{lc} = number of compliant links in a chain
- lc = link compliance content
- C = degrees of compliance, or compliance number, of a mechanism
- E = modulus of elasticity of link material
- F = degrees of freedom of a rigid-body mechanism
- F_c = degrees of freedom of a compliant mechanism
- F_r = "degenerate" degrees of freedom, or freedom number, of a compliant mechanism
- I = sectional moment of inertia of a link
- L_i = link i in a mechanism

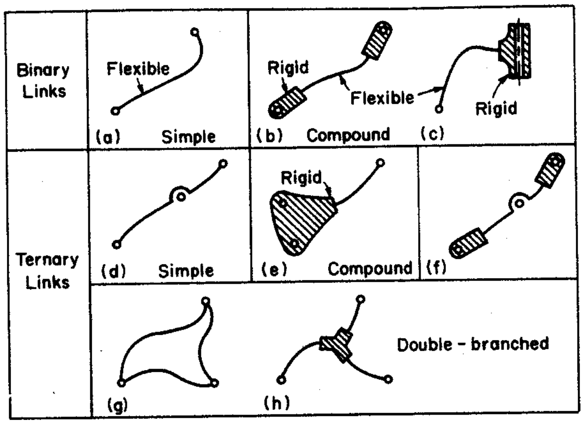

Fig. 2 Some compliant links

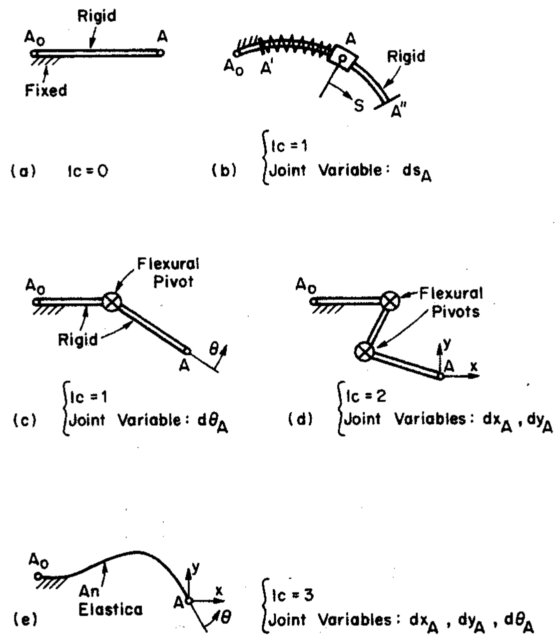

Fig. 3 Planar binary link compliance content (lc)

propriate rigid-body joint variables. All such inputs to and outputs from a compliant mechanism, on the other hand, cannot be easily transferred to nearby joints. We will treat these joints of inputs and outputs as special joints, namely "pseudojoints." There are elastic degrees of freedom associated with a pseudojoint due to the compliance of the link. They are also referred to as elastic joint variables. For example, for the planar elastica fixed at point A_o in Fig. 3(*e*), these variables for the pseudojoint A are described as dx_A, dy_A, and $d\theta_A$. For a planar compliant mechanism, the inputs and outputs at a particular joint may be selected from the rigid-body as well as elastic joint variables. For compliant mechanisms, the corresponding joint loads may also be used as the elastic joint variables describing the inputs or outputs, since the applied loads and joint displacements of the mechanism are generally functionally related.

The addition of a pseudojoint will convert a compliant simple binary link (Fig. 2(*a*) into a simple ternary one (Fig. 2*d*)). It may transform a compliant compound binary link (Fig. 2(*b*)) into compound ternary ones, as shown in Figs. 2(*e*) and 2(*f*).

Compliance Content of a Link: *lc*

When a compliant link is completely fixed at one end, unlike a rigid link, at least one elastic degree of freedom will be permitted at the other end. Since it permits relative motion between its ends, a compliant link may be viewed as a type of kinematic joint. Such joints have also been regarded as "flexural pairs" or "distributed joints" [15]. By counting the degrees of freedom of relative motion that a compliant link allows between its joints, we evaluate its "link compliance (*lc*)" content. For planar binary compliant links, *lc* is at least unity, and at most three. In Fig. 3(*e*), these are due to the elastic displacements of joint A, i.e., dx_A, dy_A, and $d\theta_A$, relative to joint A_o.

It is then convenient to classify various compliant links by their compliance content, *lc*. In Fig. 3, various types of basic planar binary links are shown, and their compliance contents quantified. For a rigid-body link (Fig. 3(*a*)), *lc* is obviously zero. Two types of compliant links, with *lc* = 1, are shown in Figs. 3(*b*) and 3(*c*). Links with higher *lc* can be obtained by joining links with lower *lc* in series, as illustrated in Fig. 3(*d*). If a link is a planar elastica (Fig. 3(*e*)), then it will have a compliance content (*lc*) equal to three. Although compliant links allow motion between their joints, this motion may by no means be unbounded. For example, in Fig. 3(*b*), we may arbitrarily prescribe that joint A only travel between points A′ and A″. Similarly, for the elastica in Fig. 3(*e*), we may argue that the motion of point A be such that it never causes the link to extend.

Ideally, an infinite number of elastic degrees of freedom are required to describe the deformation of a compliant link. In finite element methods, the deformation is assumed to be representable by finite degrees of freedom, making for approximate solution techniques. The continuum of the structure is discretized into a finite number of segments, known as elements. The degrees of freedom describe the deformations at the nodes (joints) of the elements. Confining our interest, presently, to joints where all inputs (force and displacement boundary conditions) shall be prescribed and the outputs sought, we shall ignore all of the other intermediate elastic degrees of freedom. It should be noted that this is done only to facilitate a unique and meaningful definition of the link compliance content, *lc*.

A Degree-of-Compliance Concept

The degrees of freedom of a rigid-body kinematic chain dictate the number of independent input parameters which must be controlled in order to yield desired chain configurations. Such a configuration may be described in terms of the position and orientation variables of the joint. The degrees of freedom for simpler rigid-body mechanisms may be evaluated by inspection, Grübler's criterion may be used to compute the degrees of freedom of planar rigid-body mechanisms:

$$F = 3(n_l - 1) - 2n_{j1} - n_{j2} \tag{1}$$

where n_l denotes the number of links in the mechanism, and n_{j1} and n_{j2} are the number of single-degree-of-freedom and two-degrees-of-freedom pairs, respectively.

A compliant mechanism, as discussed earlier, may consist of both rigid-body as well as flexible links, e.g., see Fig. 4. Its total response may be viewed as being compsed of rigid-body and elastic displacements. Then, the degrees of freedom of a compliant mechanism, F_c, may be defined as the summation of the degrees of freedom of the two displacement types

$$F_c = f_r + f_e \tag{2}$$

where f_r represents the degrees of freedom of the motion associated with only the rigid-body links of the mechanism. To compute f_r, we count the number of links, n_l, and the number of joints, n_{ji} (i = 1, 2), assuming all compliances (distributed or discrete) to be absent, and use equation (1). In Fig. 4, for example, when the compliant link L_5 is removed, the mechanism remaining is a rigid-body four-bar. Hence, f_r

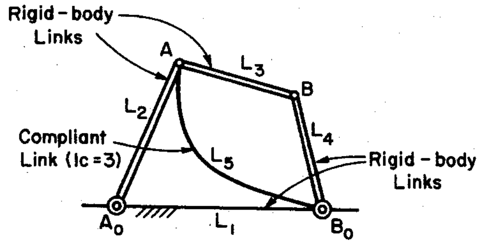

Fig. 4 A simple compliant mechanism

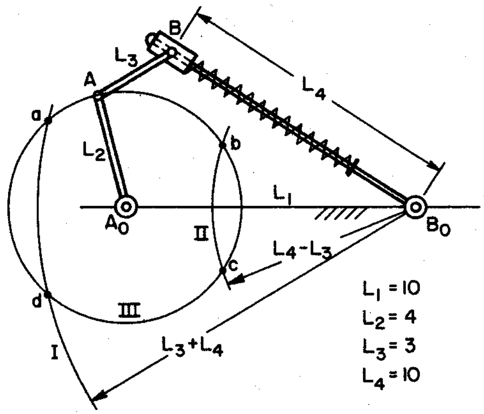

Fig. 5 Mobility region for a compliant mechanism

for this mechanism is one. The second term in equation (2), f_e, is evaluated by first fixing all rigid-body links, and then enumerating the elastic degrees of freedom of all joints, including pseudojoints. In Fig. 4, for example, these are $d\theta_{5A}$ and $d\theta_{5B_0}$. This enumeration follows from the convention of elastic displacements of a joint in Fig. 3(*e*), and the fact that no linear displacements are permitted at joints A and B_0. Hence $f_e = 2$, giving

$$F_c = 1 + 2 = 3.$$

Yet another important property of a compliant mechanism is defined to be its freedom number, F_r; this is obtained when all flexible links are regarded as stiff. This situation arises when the behavior of the compliant mechanism approaches that of a rigid-body one. These "degenerate degrees of freedom," F_r, are calculated using equation (1), with n_{ji} equaling the number of rigid-body joints, and n_l the number of all links, rigid or flexible. Again, for the mechanism in Fig. 4, $n_l = 5$ and $n_{j1} = 5$; from equation (1),

$$F_r = 3 \times (5-1) - 2 \times 6 = 0$$

Generally, for a compliant mechanism, F_r should be less than unity. This implies that the mechanism will be a structure, and the motion of the mechanism begins only when loads are applied. Moreover, if the deformations of the flexible links are within the material elastic limits, the compliant mechanism will be restored to its original state when the applied loads are removed.

When the freedom number, or degenerate degrees of freedom, F_r, for a compliant mechanism is unity or greater, the mechanism will have some rigid-body motion. While the mobility regions for rigid-body mechanisms are more readily understood [16], the mobility region for a compliant mechanism, consisting of rigid-body and elastic displacements, is much more complex. Take the simple compliant mechanism in Fig. 5, for example, where L_2 is the driving link, and L_4 the compliant link with lc equal to one. Its rigid-body mobility region can be shown [16] to correspond to the rotation of L_2 from point *a* to point *b* and from point *c* to point *d*, on the segments lying within the annular space between loci (circles) I and II. Outside this annular space, motion from the mechanism may be obtained only by deforming its flexible link L_4. If L_4 is allowed compression only, as shown in Fig. 5, the mobility region of this mechanism then corresponds to L_2 rotation between point *a* and point *d*, along the path a-b-c-d. For more complex compliant mechanisms, the mobility regions become more difficult to find. Its formal treatment is beyond the scope of this paper.

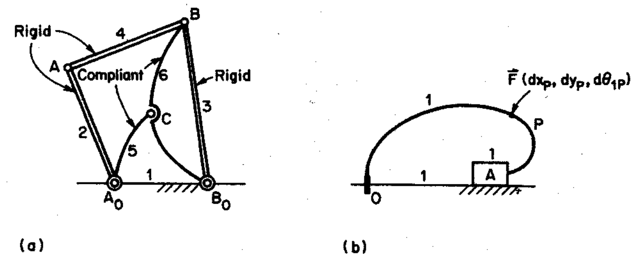

Fig. 6 Compliant mechanisms

The difference between F_c and F_r represents the degrees of freedom gained by introducing compliance into the mechanism. It would be convenient to characterize a mechanism also by its compliance content. To this end, we define the term "degrees of compliance," or the compliance number, C as

$$C = F_c - F_r \tag{3}$$

The C number is always greater than zero for compliant mechanisms. The larger the C number, the higher the deformation modes of the links that may be expected. From preceding discussions, we know that a flexible link increases the degrees of freedom of a mechanism by lc, the link compliance content. Then, for mechanisms consisting only of binary compliant links, say, and each link having the same lc

$$n_{lc} = C/lc \tag{4}$$

where n_{lc} is the number of compliant links in the mechanism.

Example 1:

Problem statement: Identify the degrees of compliance of the mechanisms shown in Figs. 6(*a*) and 6(*b*). Assume all compliant links are planar and possess a compliance content, lc, equal to 3.

In Fig. 6(*a*), when all flexible links are removed, the rigid-body degrees of freedom equal one, i.e., $f_r = 1$. To evaluate f_e, all rigid links are frozen. The elastic degrees of freedom (f_e) associated with compliant links L_5 and L_6, in accordance with the elastic displacements convention in Fig. 3(*e*), are dx_c, dy_C, $d\theta_{5A_0}$, $d\theta_{5C}$, $d\theta_{6B}$, $d\theta_{6B_0}$, and $d\theta_{6C}$. Note that point C experiences the same linear displacements dx_C and dy_C whether lying on L_5 or L_6. However, the angular displacements of L_5 and L_6 at point C, $d\theta_{5C}$ and $d\theta_{6C}$, respectively, are uniquely defined and independent of each other. Hence,

$$F_c = f_r + f_e = 1 + 7 = 8$$

The freedom number, or the degenerate degrees of freedom, F_r, for this mechanism with $n_l = 6$ and $n_{j1} = 8$, may be calculated using equation (1)

$$F_r = 3 \times (6-1) - 2 \times 8 = -1.$$

Thus, the degrees of compliance

$$C = F_c - F_r = 8 - (-1) = 9$$

The compliant mechanism shown in Fig. 6(*b*) has only one link L_1, and has only one rigid-body joint at A. An input force F is applied to the mechanism at point P. It is expressed as a function of the displacements at point P, as shown in the figure. Since inputs and outputs may be prescribed at point P, this point is treated as a pseudojoint. Since no rigid links are used, $f_r = 0$. The elastic degrees of freedom are dx_A, dx_P, dy_P and $d\theta_{1P}$, giving $f_e = 4$. We thus obtain:

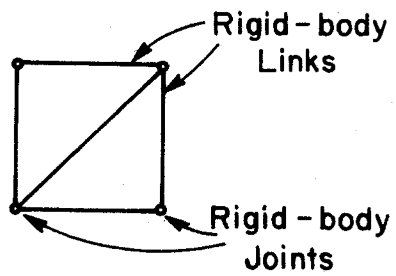

Fig. 7 A rigid-body kinematic chain

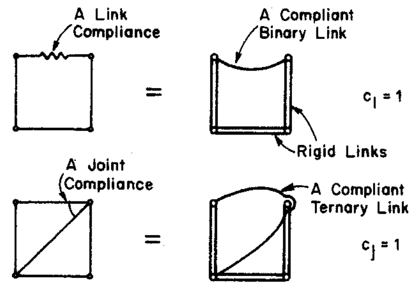

Fig. 8 Explanation of c_l and c_j

$$F_c = 0 + 4 = 4$$

Note that point O is neither a rigid-body joint nor a pseudo-joint, and requires no degrees of freedom for its description. The F_r, once again, is calculated as per equation (1)

$$F_r = 3 \times (1-1) - 2 \times 1 = -2$$

Therefore, from equation (3)

$$C = 4 - (-2) = 6.$$

In the absence of the force F in Fig. 6(*b*), it can be easily verified that the compliance number is 3. Thus, the compliance number, C, is duly related to the loading condition of the mechanism.

Significance of F_c, F_r, and C

It is well known that an n-degrees-of-freedom, rigid-body mechanism requires n independent input parameters to be controlled in order to yield deterministic configurations of the mechanism. While there is no unique equivalent definition for compliant mechanisms, the degrees of freedom, F_c, and the degenerate degrees of freedom, F_r, both serve to fulfill this need.

As per equation (2), F_c is the sum of f_r and f_e. Thus, F_c is the theoretical maximum number of inputs that can be prescribed to the compliant mechanism. This is consistent with the total degrees of freedom associated with the mechanism joints, rigid-body and elastic. Physically, the larger this number, F_c, the higher will be the number of possible modal configurations (branch solutions) achievable by the compliant mechanism.

The degenerate degrees of freedom, F_r, indicates, in the strictest sense, mobility of the entire compliant mechanism within its rigid-body mobility regions. For such mobility determination, no elastic energy is assumed to be transferred, i.e., the compliant links remain undeformed. This, therefore, implies that F_r is the theoretical minimum number of degrees of freedom that must be specified in order to yield deterministic (minimum-energy) configurations of the compliant mechanism. Physically, the lower this number, F_r, the greater will be the needed levels of elastic energy transference to obtain the desired mechanism response.

The compliance number, C, is comprised of the above-mentioned numbers F_c and F_r, as shown in equation (3). It represents the range between the defined maximum and minimum values, F_c and F_r, respectively, for the compliant mechanism. Although a concise number, C would need to be accompanied by either F_c or F_r in order that the compliant behavior of the mechanism is more effectively described.

Type Synthesis of Compliant Mechanisms

Type synthesis of rigid-body mechanisms normally deals with the selection of the mechanism type, and the number of links and joints required for the mechanism to allow a finite degrees-of-freedom motion. Many authors have contributed significantly to techniques for enumerating and identifying rigid-body mechanisms, see for instance [4, 13, 14]. A comprehensive listing of such works is beyond the scope of our research. In the area of compliant mechanisms, however, type synthesis is not yet well-defined in the literature. The initial study of type synthesis of compliant mechanisms was provided by Burns and Crossley [5], in their investigation of structural permutations of four-bar chains containing one or more flexible members. Shoup and McLarnan [7] used a different approach to discover and display flexible link mechanisms containing no rigid-body degrees of freedom. They first assume these mechanisms to possess a certain freedom number which is less than or equal to zero. Then appropriate combinations of the number of links and joints are determined from use of the Grübler's criterion. Only single-loop mechanisms were treated.

The type synthesis of compliant mechanisms discussed herein is a methodology for deriving all possible compliant mechanisms from a given rigid-body kinematic chain. Unlike the number synthesis of rigid-body mechanisms, this procedure does not require the derived mechanisms to possess the same number of links or joints as the rigid-body kinematic chain. However, it does require the shape, or the topography, of the rigid-body kinematic chain to be preserved. The type synthesis of compliant mechanisms may be accomplished in several stages. First, we shall derive all possible compliant kinematic chains from the given rigid-body one. Since these resemble the rigid-body chain in shape, each of these is termed as an "instance" of the original chain. The rigid-body chain, e.g., in Fig. 7, possesses no degrees of compliance. Compliance is introduced into the kinematic chain by (*i*) replacing rigid-body links with flexible ones, and (*ii*) imposing discrete compliances at the joints. A pictorial explanation of the link and joint compliances is provided in Fig. 8. While these compliant representations are shown to be simple binary and ternary links, respectively, they can indeed be of other forms as well. For instance, the joint compliance in Fig. 8 may be comprised of two rigid bars connected by a flexural pivot. A compliant ternary link results since the joint compliance is added where a double joint exists (Fig. 8). A joint compliance introduced at a single joint will yield a compliant binary link. For each combination of the link and joint compliances, many instances of the original kinematic chain result, as will be exemplified later.

Let c_l designate the number of rigid links replaced by flexible ones, and c_j the number of joints in the chain incorporating compliances. Such constrained joints no longer contribute to rigid-body degrees of freedom of the system, since they resist motion between the connecting links. These links, whether rigid or flexible, are now regarded as forming a single compliant link (see Fig. 3(*c*)). Constraining a joint in a mechanism in this manner reduces both the number of joints and links by one. If one continues until all rigid-body joints have been constrained, the number of joints reduces to zero and the number of links equals unity; the kinematic chain becomes fully compliant.

In enumerating possible instances for the given rigid-body

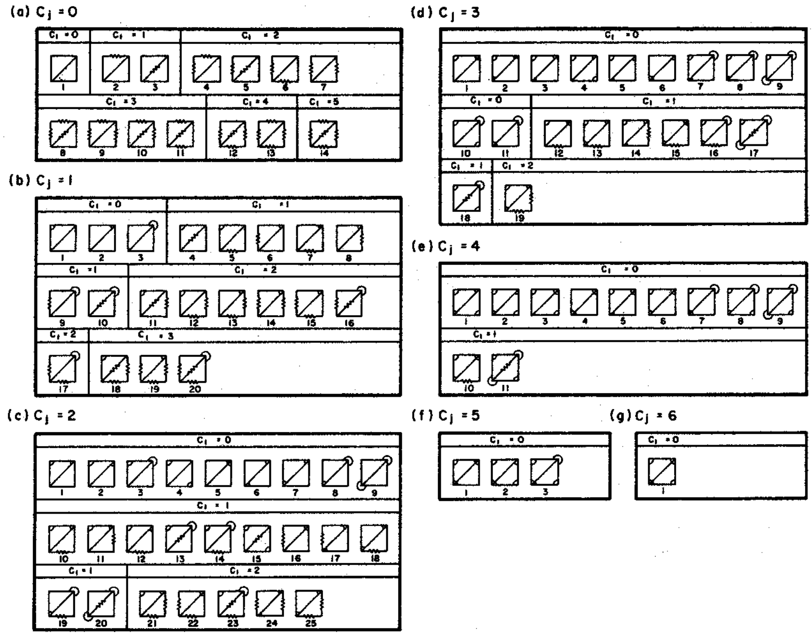

Fig. 9 Instances of the kinematic chain shown in Fig. 7

chain, in the first stage, the compliant link types need not be specified. However, this task will be accomplished in the second stage. A specific instance is chosen, in order to yield desired functions, and appropriate freedom number and link compliance content are imparted to it. Rigid-body joints are substituted with alternative kinematic pair types, if desired, to obtain different constructions.

From a particular rigid-body kinematic chain, a number of distinct mechanisms may be obtained by fixing different parts of the chain as the frame. This process, known as kinematic inversion, constitutes the third stage of type synthesis. An n-link rigid-body chain yields n kinematic inversions. In these mechanisms, the relative motions between the various links are not altered. The number of kinematic inversions for an n-link compliant chain, however, is generally greater than n. For elucidation, regard a compliant link as being composed of many (ideally infinite) rigid-body segments joined elastically at their ends, as described in previous sections. These rigid-body segments, as before, can equivalently be fixed to serve as the frame in obtaining the ideally infinite inversions. The manner in which a flexible link is fixed, therefore, will also affect the various degrees of freedom, f_r, f_e, and F_c, and the degrees of compliance C. As will be exemplified later, a more pragmatic approach is needed to characterize and yield a finite number of inversions.

Example 2:

Problem statement: Enumerate all possible instances for the rigid-body kinematic chain shown in Fig. 7.

The planar kinematic chain in Fig. 7 has 5 binary links and 6 revolute joints, giving a zero-degree-of-freedom chain. No inversion of this simple multi-loop chain possesses any rigid-body degrees of freedom. To enumerate its instances, all possible combinations of c_j and c_l are checked. Symbolic representations for link compliance (c_l) and joint compliance (c_j) are shown in Fig. 8. Any rigid link associated with either compliance type becomes part of a compliant link. The specific compliant link type selection is not considered in this stage.

The original chain (Fig. 7) essentially has $c_j = c_l = 0$. All possible instances for $c_j = 0$, with c_l varying from 0 to 5, are developed in Fig. 9(a). These 14 instances, similar to the original rigid-body chain, are all 5-link, 6-joint kinematic chains, since c_j remains at a value of zero. Fig. 9(b) shows instances with c_j equal to 1 and c_l varying from 0 to 3. They now all become 4-link, 5-joint kinematic chains. Note that of these 20 configurations, 14 have a compliant ternary link, e.g., in Figs. 9(b)(2) and 9(b)(3). This is the result of imposing a joint compliance on a double joint in the kinematic chain.

Shown in Fig. 9(c) are instances with $c_j = 2$, and c_l varies from 0 to 2. Now we have 25 instances that are 3-link, 4-joint kinematic chains. It is obvious that as c_j increases, the variability in c_l decreases. Figure 9(d) shows 19 instances corresponding to $c_j = 3$. Most of these instances are 2-link, 3-joint chains. However, there exist instances which are 3-link and 3-joint chains, i.e., Figs. 9(d)(2), 9(d)(13), and 9(d)(19). These exceptions distinguish themselves by possessing a loop entirely composed of compliant links.

As c_j is increased, the number of rigid-body joints of an instance is decreased. As stated previously, the inputs and outputs of a mechanism must be prescribed at the joints, in terms of joint variables. If the number of joints becomes too small, then pseudojoints should appropriately be introduced into the system. Figs. 9(e), 9(f), and 9(g) depict instances for a c_j value of 4, 5 and 6, respectively. When c_j equals 6, the kinematic chain becomes a fully compliant structure. Thus, for the given rigid-body kinematic chain we have found 93 distinct compliant kinematic chains, as per the proposed type synthesis methodology.

Example 3:

Problem statement: The kinematic chain in Fig. 10(a) is the instance shown in Fig. 9(b)(2). A compliant mechanism developed therefrom is illustrated in Fig. 10(b). Link L_3 is a slider, locating the joint C between links L_2 and L_3 at infinity. Remembering that the elastica 4 can have numerous representations, assume that it is represented here as a compliant simple ternary link L_4(A-D-B). Identify the kinematic inversions of this chain, and find the degrees of compliance for the resulting mechanisms.

The given kinematic chain is composed of three rigid links and one compliant link. Inversions which make use of a rigid link as the frame are easily identified. Fig. 11(a) shows the first such kinematic inversion, for which link L_1 is fixed as the

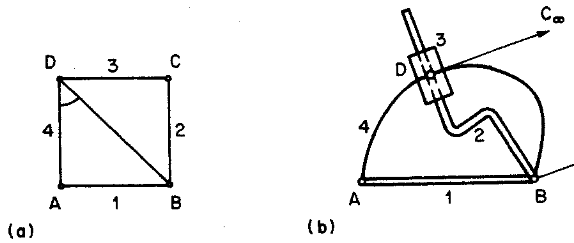

Fig. 10 A compliant kinematic chain

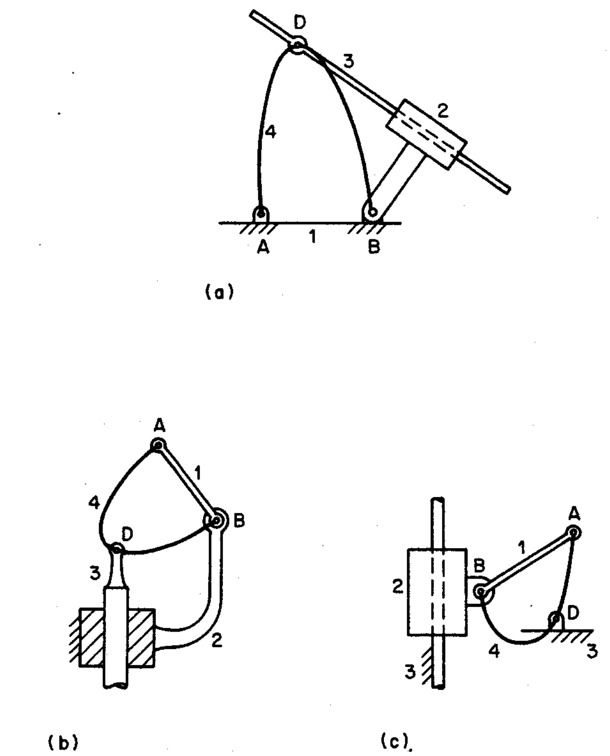

Fig. 11 Kinematic inversions by fixing a rigid link as frame

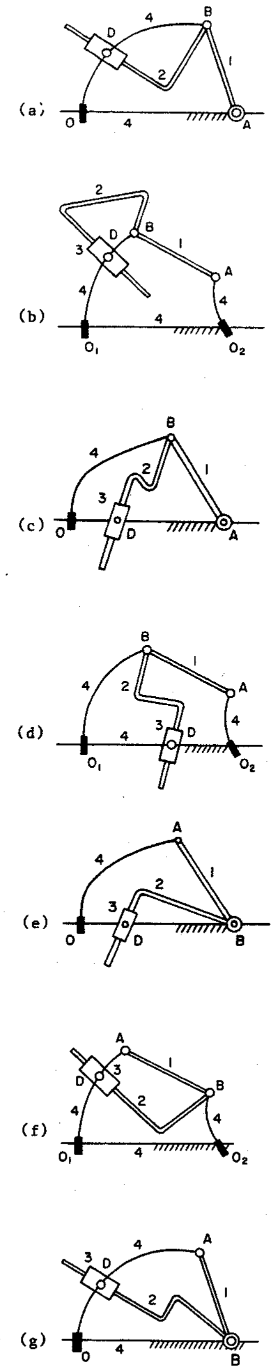

Fig. 12	f_r	f_e	F_r	C
(a)	3	2: $(d\theta_{4B}, d\theta_{4D})$	-1	6
(b)	5	3: $(d\theta_{4A}, d\theta_{4B}, d\theta_{4D})$	-1	9
(c)	1	1: $(d\theta_{4B})$	-1	3
(d)	3	2: $(d\theta_{4A}, d\theta_{4B})$	-1	6
(e)	1	1: $(d\theta_{4A})$	-1	3
(f)	5	3: $(d\theta_{4A}, d\theta_{4B}, d\theta_{4D})$	-1	9
(g)	3	2: $(d\theta_{4A}, d\theta_{4D})$	-1	6

Fig. 12 Kinematic inversions by fixing part of the compliant link as frame

frame. The second kinematic inversion is shown in Fig. 11(*b*) wherein link L_2 is fixed. The last inversion, that uses a rigid link (L_3) as frame, is illustrated in Fig. 11(*c*). For all three mechanisms shown in Fig. 11, f_r is equal to 2. The elastic degrees of freedom of the compliant link are $d\theta_{4A}$, $d\theta_{4B}$, and $d\theta_{4D}$, and give $f_e = 3$. Hence, from equation (2)

$$F_c = 2 + 3 = 5$$

From equation (1), for $n_l = 4$ and $n_{j1} = 5$, the freedom number

$$F_r = 3 \times (4-1) - 2 \times 5 = -1$$

Then for these mechanisms, the compliance number is given by equation (3)

$$C = 5 - (-1) = 6.$$

If we choose to fix a section of the compliant link L_4 to serve as the frame, as discussed previously, theoretically there will be an infinite number of kinematic inversions. These, however, can be classified into seven distinct categories, depending upon the portion of the compliant link L_4 that is fixed. As illustrated in Fig. 12, the framed portions are (*i*) between points A and D, including point A and excluding point D (Fig. 12(*a*)), (*ii*) between points A and D, excluding points A and D (Fig. 12(*b*)), (*iii*) between points A and B (on the compliant link), including points A and D, and excluding point B (Fig. 12(*c*)), (*iv*) between points A and B (on the compliant link), excluding points A and B, and including point D (Fig. 12(*d*)), (*v*) between points A and B (on the compliant link), including points B and D, and excluding point A (Fig. 12(*e*)), (*vi*) between points D and B, excluding points D and B (Fig. 12(*f*)), and (*vii*) between points D and B, excluding point D and including point B (Fig. 12(*g*)). Also shown in Fig. 12 are the corresponding values of f_r, f_e, F_r, and C. These numbers are duly altered, except F_r, by the manner of fixture of the compliant link. It should be noted that the points O, O_1, and O_2 are not joints, since there are no degrees of freedom associated with them.

Higher-Order Inversions

All the inversions enumerated in Fig. 12, by affixing one segment of the compliant ternary link at a time, will be referred to as the first-order inversions of the mechanism. Other higher-order inversions of the mechanism will be possible if more than one, non-consecutive segments are affixed. Thus, for example, if two distinct segments of the compliant link are used as frame, then second-order inversions of the mechanism result. The study of such higher-order mechanism inversions is beyond the scope of the present work.

Conclusions

Systematically developed in this paper have been fundamental concepts for quantifying the compliance content of a link (*lc*), and the various degrees of freedom, f_r, f_e, F_c, and F_r, for a compliant mechanism. F_c and F_r have been defined as the total number of degrees of freedom, rigid-body (f_r) and elastic (f_e), and the degenerate rigid-body degrees of freedom, respectively, of the mechanism. This terminology has had to be created in order to advance an important concept of the degrees of compliance, or the compliance number C, for characterizing this class of mechanisms. Numerically, this number simply represents the difference between F_c and F_r. The notion of the existence of mobility regions for compliance mechanisms, as possibly being composed of rigid-body and elastic displacements, has been briefly introduced. Finally, a systematic and tractable methodology has been presented in stages for type synthesis of compliant mechanisms. Some simple, yet effective, examples have been used to demonstrate these stages in synthesis.

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